

## Trigonometric Identities Worksheet

Given:

Fundamental  
Identities

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$\cot \theta = \frac{\cos \theta}{\sin \theta}$$

$$\csc \theta = \frac{1}{\sin \theta}$$

$$\sec \theta = \frac{1}{\cos \theta}$$

$$\cot \theta = \frac{1}{\tan \theta}$$

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$\tan^2 \theta + 1 = \sec^2 \theta$$

$$1 + \cot^2 \theta = \csc^2 \theta$$

Establish (or verify) the following identities.

1.  $\frac{1 - \tan \theta}{1 + \tan \theta} = \frac{\cot \theta - 1}{\cot \theta + 1}$

2.  $\frac{\cos^2 x}{1 + \sin x} = 1 - \sin x$

3.  $\tan \theta \sin \theta = \sec \theta - \cos \theta$

4.  $\frac{\sin \theta}{1 + \cos \theta} + \frac{1 + \cos \theta}{\sin \theta} = 2 \csc \theta$

5. If  $\tan \alpha = -\frac{4}{3}$ ,  $\frac{\pi}{2} < \alpha < \pi$  and  $\cos \beta = -\frac{1}{2}$ ,  $\pi < \beta < \frac{3\pi}{2}$ , find the exact value of:

(a)  $\sin(\alpha + \beta)$

(b)  $\tan(\alpha + \beta)$

6. Use the given angle sum and difference identities to verify the following identities:

$$\cos(A + B) = \cos A \cos B - \sin A \sin B \text{ and}$$

$$\cos(A - B) = \cos A \cos B + \sin A \sin B$$

(a)  $\cos\left(\frac{\pi}{2} - \theta\right) = \sin \theta$

(b)  $\cos(\pi + \theta) = -\cos \theta$

7. Use the angle sum identity  $\sin(A + B) = \sin A \cos B + \cos A \sin B$  and  $\sin(A - B) = \sin A \cos B - \cos A \sin B$  to verify the following identities:

(a)  $\sin\left(\frac{\pi}{2} - \theta\right) = \cos \theta$

(b)  $\sin(\pi - \theta) = \sin \theta$

8. Use one of the Half-Angle formulas to find the exact value of

(a)  $\cos 15^\circ$

(b)  $\tan\left(\frac{\pi}{12}\right)$

9. If  $\cos \theta = \frac{4}{5}$  and  $0 < \theta < \frac{\pi}{2}$ , find the exact value of

(a)  $\sin(2\theta)$

(b)  $\cos(2\theta)$

(c) the quadrant in which  $2\theta$  lies