Warm - Up

Find the product of $z_1 * z_2$ in trigonometric for and standard form

$z_1 = 1 - i$ $z_2 = \sqrt{3} + i$

$(1-i)(\sqrt{3}+i)$

$\sqrt{3}+i-i\sqrt{3}-i^2$

$\sqrt{3}+1+i-i\sqrt{3}$

$2.7-.7i$

$r=\sqrt{\sqrt{3}^2+1^2}=2$

$\theta=\frac{\pi}{6}$

$r=\sqrt{1^2+-1^2}$

$r=\sqrt{2}$

$\theta=\tan^{-1}\left(\frac{-1}{1}\right)$

$\theta=-\frac{\pi}{4}$

$2\sqrt{2}\left(\cos\frac{\pi}{6}-\frac{\pi}{4}+i\sin\frac{\pi}{6}-\frac{\pi}{4}\right)$

$2\sqrt{2}\left(\cos-\frac{\pi}{12}+i\sin-\frac{\pi}{12}\right)$

SWBAT take roots of complex numbers

Agenda:

- Warm-Up
- Discuss Last Class
- Learn De Moivre's Theorem
- Exit Card

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HW Questions from Tuesday?

Lets take z^2

$r(\cos\theta + i\sin\theta)\, r(\cos\theta + i\sin\theta)$

$r^2(\cos\theta\cos\theta + \cos\theta i\sin\theta + i\sin\theta\cos\theta + i^2\sin\theta\sin\theta$

$r^2(\cos\theta\cos\theta - \sin\theta\sin\theta + \cos\theta i\sin\theta + i\sin\theta\cos\theta)$

$r^2(\cos(\theta+\theta) + i(\sin\theta+\theta)$

$\boxed{r^2(\cos 2\theta + i\sin 2\theta)}$

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$r^3(\cos 3\theta + i\sin 3\theta)$

Now Lets take z^3

De Moivre's Theorem

Let $z = r(\cos\theta + i\sin\theta)$ and let n be a positive integer. Then

$$z^n = [r(\cos\theta + i\sin\theta)]^n = r^n(\cos n\theta + i\sin n\theta)$$

Example: Use DeMoivre's theorem to calculuate

$(1 + i)^4$

$r = \sqrt{1^2 + 1^2} = \sqrt{2}$

$\tan\theta = \frac{1}{1}$

$\theta = \frac{\pi}{4}$

$\sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$

$\sqrt{2}^4\left(\cos \cancel{4}\cdot\frac{\pi}{\cancel{4}} + i\sin \cancel{4}\cdot\frac{\pi}{\cancel{4}}\right)$

$4(\cos\pi + i\sin\pi)$

$4(-1 + i0)$

-4

Example: Use DeMoivre's theorem to calculuate

$-\frac{1}{2} + \frac{\sqrt{3}}{2}i$

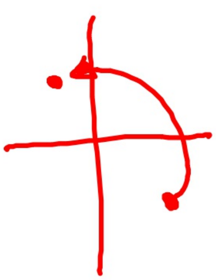
$$\left(\frac{-1+\sqrt{3}i}{2}\right)^6$$

$r = \sqrt{\left(-\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} = 1$

$\theta = \tan^{-1}\left(\frac{\frac{\sqrt{3}}{2}}{-\frac{1}{2}}\right) = -60$

$+180$

120



$1^6(\cos 6\cdot 120 + i\sin 6\cdot 120)$

$1(\cos 720 + i\sin 720)$

$1(1 + i0)$

1

If z = r(cos + isin), the nth distinct complex numbers

$$\sqrt[n]{r}\left(cos\frac{\theta + 2\pi k}{n} + isin\frac{\theta + 2\pi k}{n}\right)$$

where k = 0, 1, 2, . . . n - 1, are the nth roots of the complex number z.

Find the fourth roots of

$$z = 5(\cos(\pi/3) + i\sin(\pi/3))$$

$$z_1 = \sqrt[4]{5}\left(\cos\frac{\pi/3 + 2\pi 0}{4} + i\sin\frac{\pi/3 + 2\pi 0}{4}\right)$$

$$* \; z_1 = \sqrt[4]{5}\left(\cos\frac{\pi}{12} + i\sin\pi/12\right)$$

$$z_2 = \sqrt[4]{5}\left(\cos\frac{\pi/3 + 2\pi 1}{4} + i\sin\frac{\pi/3 + 2\pi 1}{4}\right)$$

$$* \; z_2 = \sqrt[4]{5}\left(\cos\frac{7\pi}{12} + i\sin\frac{7\pi}{12}\right)$$

$$* \; z_3 = \sqrt[4]{5}\left(\cos\frac{13\pi}{12} + i\sin\frac{13\pi}{12}\right)$$

$$* \; z_4 = \sqrt[4]{5}\left(\cos\frac{19\pi}{12} + i\sin\frac{19\pi}{12}\right)$$

Find the cube roots of -1.

Exit Card

Use DeMoivre's theorem to find:

$$\left[3\left(\cos\frac{3\pi}{2} + i\sin\frac{3\pi}{2}\right)\right]^5$$

Put your answer in standard form a + bi