Warm - Up

$i^1 =$ i

$i^2 = -1$

$i^3 = -i$

$i^4 = 1$

$i^5 =$ i

$i^6 = -1$

$i^7 = -i$

$i^8 = 1$

SWBAT represent complex numbers in the complex plane

Agenda

- Warm - Up
- Refresh on complex numbers
- Complex Plane
- Trigonometry with complex numbers
- Exit Card

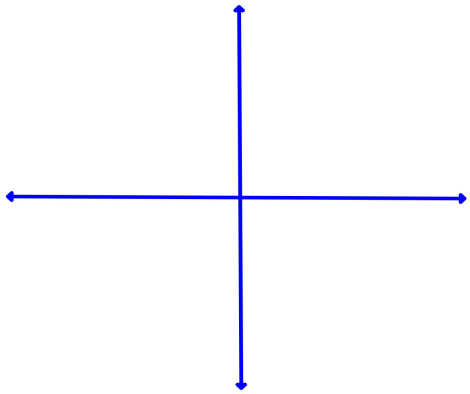
HW Book page 559 1 - 30 odd

$\sqrt{-9} =$ $\sqrt{9(-1)} = \sqrt{9}\sqrt{-1}$ $3i$

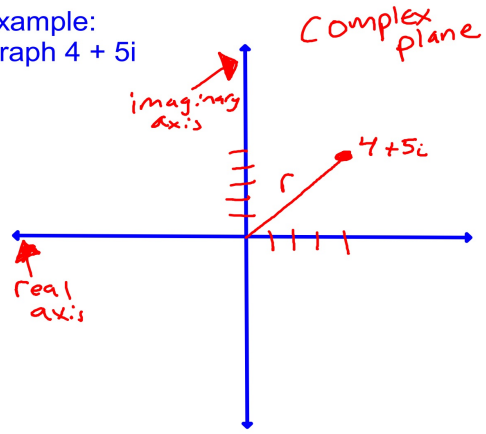
$2i + 3i = 5i$

$(3i)(4i) = 12i^2$ -12

Complex Plane



Example:
Graph 4 + 5i



Absolute Value of a Complex Number

The absolute value or modulus of a complex number z = a + bi is

$$|z| = |a + bi| = \sqrt{a^2 + b^2}$$

In the complex plane, |a + bi| is the distance of a + bi from the origin

The trigonometric form of the complex number z = a + bi is

$$z = r(\cos\theta + i \sin\theta)$$

where $a = r\cos\theta$, $b = r\sin\theta$, $r = \sqrt{a^2 + b^2}$, $\tan\theta = b/a$

Find the rectangular form of

$$z = 2\sqrt{3}(\cos 120^\circ + i \sin 120^\circ)$$

$2\sqrt{3}\left(-\frac{1}{2} + i\frac{\sqrt{3}}{2}\right)$

$2\sqrt{3}\left(-\frac{1}{2}\right) + 2\sqrt{3}(i)\left(\frac{\sqrt{3}}{2}\right)$

$-\sqrt{3} + 3i$

Find the rectangular form of

(b) $w = 2(\cos 3.7 + i \sin 3.7)$

$2\cos 3.7 + i2\sin 3.7$

$-1.7 \quad -1.06i$

Find the trigonometric form with $0 \le \theta \le 2\pi$ for the complex number

$1 - \sqrt{3}i$

$r = \sqrt{1^2 + (-\sqrt{3})^2}$

$r = \sqrt{4}$

$r = 2$

$\tan\theta = \frac{-\sqrt{3}}{1}$

$\theta = \tan^{-1}\left(\frac{-\sqrt{3}}{1}\right)$

$\theta = -\frac{\pi}{3}$

$\theta = \frac{5\pi}{3}$

$2\left(\cos\frac{5\pi}{3} + i\sin\frac{5\pi}{3}\right)$

Find the trigonometric form with $0 \le \theta \le 2\pi$ for the complex number

$-3 - 4i$

$r = \sqrt{-3^2 + -4^2}$

$r = \sqrt{25}$

$r = 5$

$\theta = \tan^{-1}\left(\frac{-4}{-3}\right)$

$\theta = 53$

$\theta = 233.\frac{\pi}{180}$

$\theta = 4.07$

$5(\cos 4.07 + i\sin 4.07)$

Lets multiply $z_1 * z_2$

$z_1 = r_1(\cos\theta_1 + i\sin\theta_1) \quad z_2 = r_2(\cos\theta_2 + i\sin\theta_2)$

$(r_1\cos\theta_1 + r_1 i\sin\theta_1)(r_2\cos\theta_2 + r_2 i\sin\theta_2)$

$r_1\cos\theta_1 r_2\cos\theta_2 + r_1\cos\theta_1 r_2 i\sin\theta_2 + r_1 i\sin\theta_1 r_2\cos\theta_2 + r_1 i\sin\theta_1 r_2 i\sin\theta_2$

$r_1 r_2(\cos\theta_1\cos\theta_2 + i\cos\theta_1\sin\theta_2 + i\sin\theta_1\cos\theta_2 + i^2\sin\theta_1\sin\theta_2)$

$r_1 r_2(\cos\theta_1\cos\theta_2 - \sin\theta_1\sin\theta_2 + i\cos\theta_1\sin\theta_2 + i\sin\theta_1\cos\theta_2)$

$r_1 r_2[(\cos\theta_1\cos\theta_2 - \sin\theta_1\sin\theta_2) + i(\cos\theta_1\sin\theta_2 + \sin\theta_1\cos\theta_2)]$

$r_1 r_2(\cos(\theta_1+\theta_2) + i\sin(\theta_1+\theta_2))$

Product and Quotient of Complex Numbers

Let $z_1 = r_1(\cos\theta_1 + i\sin\theta_1)$ and $z_2 = r_2(\cos\theta_2 + i\sin\theta_2)$. Then

1) $z_1 z_2 = r_1 r_2[\cos(\theta_1 + \theta_2) + i\sin(\theta_1 + \theta_2)$

2) $\frac{z_1}{z_2} = \frac{r_1}{r_2}[\cos(\theta_1 - \theta_2) + i\sin(\theta_1 - \theta_2)$

Express the product of z_1 and z_2

Let $z_1 = 25\sqrt{2}\left(\cos\frac{-\pi}{4} + i\sin\frac{-\pi}{4}\right)$ and $z_2 = 14\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$

Express the quotient of z_1 / z_2

Let $z_1 = 2\sqrt{2}(\cos 135° + i\sin 135°)$ and $z_2 = 6(\cos 300° + i\sin 300°)$

Find the product of z_1 * z_2 in trigonometric for and standard form

$z_1 = 3 - 2i$ $z_2 = 1 + i$

Exit Card