

Warm - Up

Given $\mathbf{v} = \langle -3, -2, 5 \rangle$, determine which vectors are parallel to \mathbf{v} .

✓ $\langle 6, 4, -10 \rangle$ $\frac{-3}{6} \quad \frac{-2}{4} \quad \frac{5}{-10} \quad C = -1/2$

$\langle -3/2, -1, -5/2 \rangle$ $\frac{-3}{-3/2} \quad \frac{-2}{-1} \quad \frac{5}{-5/2} \quad \times$

✓ $\langle 2, 4/3, -10/3 \rangle$

$$\frac{-3}{2} \quad \frac{-2}{4/3} \quad \frac{5}{-10/3} \quad \frac{-2}{-2/3} \quad \frac{3}{4}$$

HW

#6 $3x - 9y = 19$ $x = 6$ $y = -1/9$
 $3x - 9y = 11$

$$\frac{3x - 9y - 11}{A \quad B \quad C} = C$$

$$\frac{|3(6) - 9(-1/9) - 11|}{\sqrt{3^2 + (-9)^2}}$$

$$\begin{array}{r} 3x - 9y = 19 \\ -3x \quad -3x \\ \hline \end{array}$$

$$\frac{-9y = -3x + 19}{-9} \quad \frac{-3x + 19}{-9}$$

$$y = \frac{3x - 19}{9}$$

$y =$, \downarrow 2nd, Table

$$\frac{8}{\sqrt{90}}$$

$$\langle 15, 1 \rangle \quad \langle -20, 1 \rangle$$

$$\cos \theta = \frac{u \cdot v}{|u||v|}$$

$$\cos \theta = \frac{\langle 15, 1 \rangle \cdot \langle -20, 1 \rangle}{|\langle 15, 1 \rangle| |\langle -20, 1 \rangle|}$$

$$= \frac{15 \cdot -20 + 1 \cdot 1}{\sqrt{15^2 + 1^2} \cdot \sqrt{-20^2 + 1^2}}$$

$$= \frac{-299}{\sqrt{226} \cdot \sqrt{401}}$$

$$= \frac{-299}{301.84}$$

$$173.3^\circ$$

SWBAT write vector and parametric equations of lines and graph parametric equations

Agenda:

Warm - Up

Refresh equation of a line

Parametric equations

Exit card

Raindrops are falling and blown by the wind.

This gives us a horizontal and a vertical component.

However, now we want to talk about time

The graph of the ordered pair (x, y) where

$$x = f(t), y = g(t)$$

are functions ~~defined on~~^{on} an interval I of t values is a parametric curve. The equations are parametric equations of the curve, the variable t is a parameter.

Graphing Parametric Equations

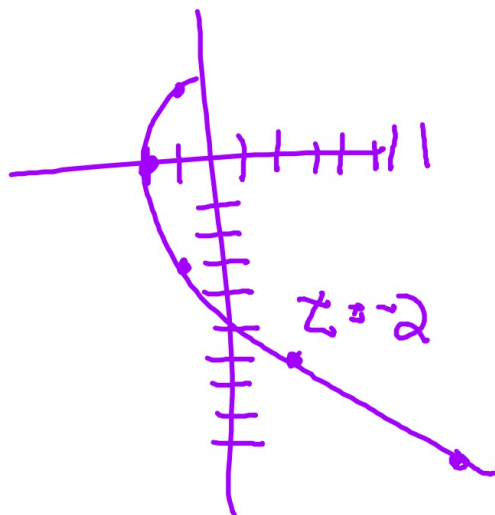
Graph the parametric equations

$$x = t^2 - 2$$

$$y = 3t$$

on the interval $-3 \leq t \leq 1$

x	y	t
7	-9	-3
2	-6	-2
-1	-3	-1
-2	0	0
-1	3	1



A line through $P_1(x_1, y_1)$ and parallel to the vector $\mathbf{a} = \langle a_1, a_2 \rangle$ has vector equation:

~~$$\mathbf{r} = \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x_1 \\ y_1 \end{pmatrix} + t \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$$~~

and parametric equations

$$x = x_1 + ta_1$$

$$y = y_1 + ta_2$$

Example:

$$P(-2, 1), \mathbf{a} = \langle 3, -4 \rangle$$

$$x = -2 + 3t$$

$$y = 1 - 4t$$

Example:

Write the parametric equations of the line

$$y = (-1/2)x + 3$$

$$\rightarrow \mathbf{a} = \langle 2, -1 \rangle \quad P(6, 0)$$

$$\begin{array}{rcl} 0 & = & -\frac{1}{2}x + 3 \\ -3 & & -3 \end{array}$$

$$-3 = -\frac{1}{2}x$$

$$6 = x$$

$$x = 6 + 2t$$

$$y = 0 - 1t$$

$$y = -t$$

Now take

$$x = x_1 + ta_1$$

$$y = y_1 + ta_2$$

and solve for t

$$t = \frac{x - x_1}{a_1}$$

$$t = \frac{y - y_1}{a_2}$$

Therefore, the parametric equations of a line passing through two points $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$

$$x = x_1 + (x_2 - x_1)t$$

$$y = y_1 + (y_2 - y_1)t$$

Example:

Write the parametric equations of the line through the points A(-2, 0) and B(2, 2).

$$x = -2 + 4t$$

$$y = 2t$$

Eliminating the Parameter

Eliminate the parameter and identify the graph of the parametric curve

$$x = 1 - 2t$$

$$y = 2 - t$$

$$y = 2 - t$$

$$t + y = 2$$

$$t = 2 - y$$

$$x = 1 - 2(2 - y)$$

$$x = 1 - 4 + 2y$$

$$x = -3 + 2y$$

$$\frac{x+3}{2} = \frac{2y}{2}$$

$$\frac{1}{2}x + \frac{3}{2} = y$$

Eliminating the Parameter

Eliminate the parameter and identify the graph of the parametric curve

$$\begin{aligned}
 x &= t^2 - 2 \\
 y &= 3t
 \end{aligned}$$

Handwritten work:

$$\begin{aligned}
 x+2 &= \frac{y^2}{9} \cdot 9 \\
 \sqrt{9(x+2)} &= \sqrt{y^2} \\
 \pm 3\sqrt{x+2} &= y
 \end{aligned}$$

$$t = \frac{y}{3}$$

$$x = \left(\frac{y}{3}\right)^2 - 2$$

$$x = \frac{y^2}{9} - 2$$

$$\sqrt{9 \cdot 16} = \sqrt{9} \cdot \sqrt{16}$$

Eliminate the parameter and identify the graph of the parametric curve

$$\begin{aligned}
 x &= 2\cos(t) \\
 y &= 2\sin(t)
 \end{aligned}
 \quad 0 \leq t \leq 2\pi$$

$$\begin{aligned}
 x^2 &= 4\cos^2 t \\
 y^2 &= 4\sin^2 t \\
 x^2 + y^2 &= 4\cos^2 t + 4\sin^2 t \\
 x^2 + y^2 &= 4(\cos^2 t + \sin^2 t) \\
 x^2 + y^2 &= 4
 \end{aligned}$$

Eliminate the parameter and identify the graph of the parametric curve

$$\begin{aligned} x &= 3\cos(4t) + 5 \\ y &= 3\sin(4t) - 4 \end{aligned} \quad 0 \leq t \leq 2\pi$$

$$\frac{2\pi}{4} = \frac{\pi}{2}$$

Circle

$$r = 3$$

4 times around
Circle

Find the parametrization of the line through the points $A = (-2, 3)$ and $B = (3, 6)$

Now, find a parametrization of this line segment.

Quiz Outline for Friday

Parts

- 1) Entry Card from last class
- 2) Unit Vector
- 3) Vector Word Problem
- 4) Angle Between two Vectors
- 5) Vector Projection
- 6) Force

Exit Card

Eliminate the parameter.

$$x = t - 4$$

$$y = t^2 + t - 6$$

$$t = x + 4$$

$$y = (x+4)^2 + x+4 - 6$$

$$y = x^2 + \underline{8x} + \underline{16} + \underline{x} + \underline{4} - \underline{6}$$

$$x^2 + 9x + 14$$

