

Warm - Up

A force of 50 lb acts on an object at an angle of 45° while a second force of 75 lb acts on the same object at an angle of -30° . Find the magnitude and direction of the resultant vector.

SWBAT multiply vectors using the dot product

Agenda:

- Warm-Up

- HW Q's

- Learn Dot Product

- Small Self discovery activity

- Exit Card

Dot Product: Given $u = \langle u_1, u_2 \rangle$ and $v = \langle v_1, v_2 \rangle$

$$\text{then } u \bullet v = u_1 v_1 + u_2 v_2$$

Properties of the Dot Product

Let u , v , and w be vectors and let c be a scalar:

1) $u \bullet v = v \bullet u$

2) $u \bullet u = |u|^2$

3) $0 \bullet u = 0$

4) $u(v + w) = uv + uw$

5) $(cu)v = u(cv) = c(uv)$

Find the dot product

a) $\langle 3, 4 \rangle \cdot \langle 5, 2 \rangle$

b) $\langle 1, -2 \rangle \cdot \langle -4, 3 \rangle$

c) $(2i - j) \cdot (3i - 5j)$

Use the dot product to find the length of the vector $b = \langle 7, 3 \rangle$

Angle Between Two Vectors

If θ is the angle between the nonzero vectors of u and v , then

$$\cos \theta = \frac{u \bullet v}{|u||v|}$$

$$\text{and } \theta = \cos^{-1} \frac{u \bullet v}{|u||v|}$$

Find the angle between the vector $\langle 2, 3 \rangle$ and $\langle -2, 5 \rangle$

Get into small groups and take 10 - 15 minutes to answer the front side of the worksheet.

Be ready to share your findings.

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What did we find out perpendicular lines?

If vector \mathbf{u} and \mathbf{v} are perpendicular, that is, if the angle between them is 90° , then

$$|\mathbf{u}||\mathbf{v}|\cos 90^\circ = 0$$

Orthogonal Vectors

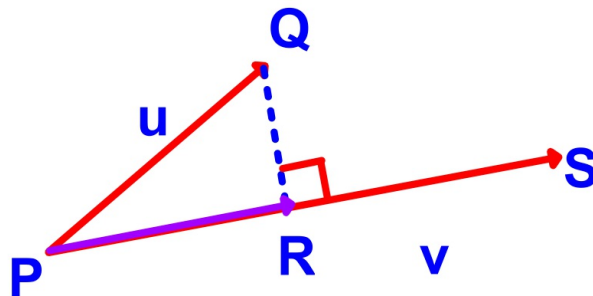
The vectors \mathbf{u} and \mathbf{v} are orthogonal IFF $\mathbf{u} \bullet \mathbf{v} = 0$

Orthogonal and Perpendicular

Basically the same except with the zero vector

zero vector is not perpendicular to any thing but is orthogonal to everything

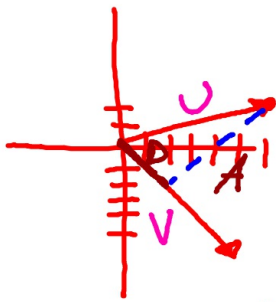
Vector Projections



if \mathbf{u} and \mathbf{v} are nonzero vectors, the projection of \mathbf{u} onto \mathbf{v} is

$$\text{proj}_{\mathbf{v}} \mathbf{u} = \frac{\mathbf{u} \cdot \mathbf{v}}{|\mathbf{v}|^2} \mathbf{v}$$

Find the vector projection of $\mathbf{u} = \langle 6, 2 \rangle$ onto $\mathbf{v} = \langle 5, -5 \rangle$. Then write \mathbf{u} as the sum of two orthogonal vectors, one of which is $\text{proj}_{\mathbf{v}} \mathbf{u}$.



$$\begin{aligned} \text{proj}_{\mathbf{v}} \mathbf{u} &= \frac{(\langle 6, 2 \rangle \cdot \langle 5, -5 \rangle)}{(\sqrt{5^2 + (-5)^2})^2} \langle 5, -5 \rangle \\ &= \frac{20}{50} \langle 5, -5 \rangle \\ &= \langle 2, -2 \rangle \end{aligned}$$

$$\begin{aligned} \mathbf{u} &= \mathbf{D} + \mathbf{A} \\ \langle 6, 2 \rangle &= \langle 2, -2 \rangle + \mathbf{A} \\ \langle 4, 4 \rangle &= \mathbf{A} \end{aligned}$$

Juan is sitting on a sled on the side of a hill inclined at 45° . The combined weight of Juan and the sled is 140 lbs. What force is required for Rachel to keep the sled from sliding down the hill.

Work:

If \mathbf{F} is a constant force whose direction is the same as the direction of AB , then the work W done by \mathbf{F} in moving an object from A to B is

$$W = |\mathbf{F}||AB|$$

If \mathbf{F} is a constant force in any direction, then the **work** W done by \mathbf{F} in moving an object from A to B is

$$\begin{aligned} W &= \mathbf{F} \bullet \mathbf{AB} \\ &= |\mathbf{F}||AB|\cos\theta \end{aligned}$$

Example

Find the work done by a 10 pound force acting in the direction $\langle 1, 2 \rangle$ in moving an object 3 ft from $(0, 0)$ to $(3, 0)$.