

Warm - Up

Entry Card, clear your desk.

SWBAT add vectors and find the components of vectors.

Agenda:

- Quiz
- Vector addition
- Unit vectors
- Exit Card

HW - Book Page 511 #5 - 41 odd, 50 and 52

Any two arrows with the same length and pointing in the same direction represent the same vector.

**Head Minus Tail Rule**

If an arrow has initial point  $(x_1, y_1)$  and terminal point  $(x_2, y_2)$ , it represents the vector

$$\langle x_2 - x_1, y_2 - y_1 \rangle$$

Example:

Show that the arrow from  $A = (-4, 2)$  to  $B = (-1, 6)$  is equivalent to the arrow from  $X = (2, -1)$  to  $Y = (5, 3)$

Magnitude:

If  $\mathbf{v}$  is represented by the arrow from  $(x_1, y_1)$  to  $(x_2, y_2)$ , then

$$|\mathbf{v}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Example:

Find the magnitude of the vector  $\mathbf{v}$  represented by PQ, where  $P = (-3, 4)$  and  $Q = (-5, 2)$

Vector Addition and Scalar Multiplication:

Let  $\mathbf{u} = \langle u_1, u_2 \rangle$  and  $\mathbf{v} = \langle v_1, v_2 \rangle$  be vectors and let  $k$  be a real number (**scalar**). the sum of the vectors  $\mathbf{u}$  and  $\mathbf{v}$  is the vector:

$$\mathbf{u} + \mathbf{v} = \langle u_1 + v_1, u_2 + v_2 \rangle$$

The product of the scalar  $k$  and the vector  $\mathbf{u}$  is:

$$k\mathbf{u} = k\langle u_1, u_2 \rangle = \langle ku_1, ku_2 \rangle$$

Example:

Let  $\mathbf{u} = \langle -1, 3 \rangle$  and  $\mathbf{v} = \langle 4, 7 \rangle$ . Find the component form of the following vectors:

(a)  $\mathbf{u} + \mathbf{v}$

$$\begin{aligned} & -1 + 4, 3 + 7 \\ & \langle 3, 10 \rangle \end{aligned}$$

(b)  $3\mathbf{u}$

$$\begin{aligned} & 3\langle -1, 3 \rangle \\ & \langle -3, 9 \rangle \end{aligned}$$

(c)  $2\mathbf{u} + -1\mathbf{v}$

$$\begin{aligned} & \langle -2, 6 \rangle + \langle -4, -7 \rangle \\ & \langle -6, -1 \rangle \end{aligned}$$

### Unit Vector:

A vector  $\mathbf{u}$  with a length  $|\mathbf{u}| = 1$  is a **unit vector**.  
If  $\mathbf{v}$  is not the zero vector then the vector

$$\mathbf{u} = \frac{\mathbf{v}}{|\mathbf{v}|} = \frac{1}{|\mathbf{v}|} \mathbf{v}$$

is a **unit vector** in the direction of  $\mathbf{v}$

### Example:

Find a unit vector in the direction of  $\mathbf{v} = \langle -3, 2 \rangle$   
and verify that it has length 1.

Standard unit vectors:

$$\mathbf{i} = \langle 1, 0 \rangle \text{ and } \mathbf{j} = \langle 0, 1 \rangle$$

Any vector  $\mathbf{v}$  can be written as an expression in terms of the standard unit vectors

$$\begin{aligned}\mathbf{v} &= \langle a, b \rangle \\ &= \langle a, 0 \rangle + \langle 0, b \rangle \\ &= a\langle 1, 0 \rangle + b\langle 0, 1 \rangle \\ &= a\mathbf{i} + b\mathbf{j}\end{aligned}$$

**Direction angle** is the angle  $\theta$  that  $\mathbf{v}$  makes with the positive x - axis.

### Resolving the Vector:

if  $\mathbf{v}$  has direction angle  $\theta$ , the components of  $\mathbf{v}$  can be computed using the formula

$$\mathbf{v} = \langle |\mathbf{v}|\cos\theta, |\mathbf{v}|\sin\theta \rangle$$

Example:

Find the components of the vector  $\mathbf{v}$  with direction angle  $105^\circ$  and magnitude of 6.

Example:

Find the magnitude and direction angle of:

$$\mathbf{u} = \langle 3, 2 \rangle$$

Quick Note:

The **velocity** of a moving object is a vector because velocity has both magnitude and direction. The magnitude of velocity is **speed**

Example:

A DC - 10 jet aircraft is flying on a bearing of  $65^\circ$  at 500mph. Find the component form of the velocity of the airplane.

### Exit Card

Let  $P = (-2, 2)$  and  $Q = (3, 4)$ . Find the component form and magnitude of the vector  $\overrightarrow{QP}$

$$|v| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$